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PHOTOELASTIC MODELLING OF DESTRUCTION PROCESSES CONSIDERING THE CREEP

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ABSTRACT

The generalized theory of damage accumulation was used in the damage tensor, based on the true strain, which is defined by a polarization optical method has been constructed. It allows us to find time before the destruction for various deformation processes on the kernels of a damage tensor found from experiences on natural samples. However, there are no effective methods of a quantitative evaluation of damage of viscoelastic materials under the influence of loading, especially in the field of their plastic deformations, by means of experimental nondestructive methods.

KEYWORDS: *Damage Tensor, Deformation, Creep, Viscoelastic, Polarization Optical Method*

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INTRODUCTION

The polarization optical method of the intense deformed state research is used successfully at the decision of static and mechanic dynamic tasks of a deformable solid body [1-3]. The generalized theory of damage accumulation which is offered in work [4] assumes the existence of two damage types – from separation and shift. The first damage type is found experimentally in polymers [5] at X-ray diffraction on these damages and represents the submicrocracks of a disk-shaped form which sizes fluctuate within $10 – 10^3$Å and don't depend on the size of loading, temperature and test time, being the characteristic of this material. Submicrocracks are practically absent in not loaded material and appear right after the loading, perpendicular to the stretching tension. Their concentration grows in time, reaching $10^{12} – 10^{16}$ cm^{-3} at the moment before destruction and defines material deformation properties. A growth rate of damage concentration significantly depends on loading and temperature.

MATHEMATICAL FORMULATION OF THE PHYSICAL PROBLEM

The relative concentration of damages $\rho = \frac{N_t}{N_p}$ where $N_t, N_p -$ current and limit concentration, changes in limits is given as

$$0 < \rho < 1 \tag{1}$$

The majority of destruction criteria is expressed through tension, however, such an approach is inconvenient during the modeling as all experimental methods of research directly allow to define only deformations. To pass to tension, it is necessary to resort to those dependencies connecting tension with deformations which follow from various theories having limited application. In a photo elasticity method, the optical anisotropy which is shown in the polarized light in the form of two-refraction depends on a relative positioning of optically sensitive material macromolecule elements and therefore contacts deformations by a linear ratio.

$$m(t)=\int_0^t E(t-\tau)d\,\varepsilon(t) \tag{2}$$

Where *m(t)* – the order of an interferential strip; $E(t)$ - the kernel or function of memory which can be defined in experiences on a relaxation [6] and $\varepsilon(t)$ – a difference of the main deformations in the plane, normal to the direction of raying

$$\varepsilon(t)=\varepsilon_1(t)-\varepsilon_2(t) \tag{3}$$

Equation (3) which is the sample from optically sensitive material has undergone deformation up to the size thereby transforming to

$$\varepsilon(t)=\varepsilon_o h(t) \tag{4}$$

Where $\dot{\varepsilon}(t)$ =const; $h(t)$ – single Heaviside function,

$$h(t)=\{\underline{1} \text{ if } \mathrm{h(t)}=\begin{cases}1 \; at\, t\geq 0\\ 0 \; at\; t<0\end{cases} \tag{5}$$

then at $t>0$, the orders of strips will decrease, decompress. Substituting equation (3) in (1) and on the property of Heaviside function [4] we will receive:

$$m(t)=\varepsilon_0 E(t), \tag{6}$$

From equ. (6) we have

$$E(t)=\frac{m(t)}{\varepsilon_0} \tag{7}$$

here dependence of deformations on a strip order can be written down similarly

$$\varepsilon(t)=\int_0^t M(t-\tau)d\;m(t) \tag{8}$$

where the kernel of *M (t)* is kernel resolvent *E(t)* and can be found by ways known from the theory of the integrated equations.

Thus, it is possible to find differences of the main deformations in a point with the known order of interferential strips using the expression (7). Separate determination of the main deformation sizes can be carried out by means of various experimental methods or a numerical method with the attraction of the equations of deformation compatibility [5]. Following [6] we will construct a damage tensor on the basis of deformations

$$p_{ij} = \int_0^t [\psi(t-\tau)d\varepsilon_{ij}(\tau) + \delta_{ij}\psi_2(t-\tau)d\varepsilon_{kk}(\tau)], \tag{9}$$

where ε_{ij} – true deformations; ψ, ψ_2 - the kernels defined in experiences on the destruction of material samples at the set simplest deformation laws. True deformations at the accepted level of the microscopic description of body material structure can be determined by a photoelasticity method with an observance of deformation processes similarity conditions. We will note that tensor representation of the saved-up damages was used repeatedly at the solution of deformable body stability problems [7], on the basis of structural approach with application of a power method the continual model of the connected process of deformation and body micro damage ability which is considered as elastic and fragile isotropic materials is under construction.

In (9) criteria of destruction invariant sizes are p_1, p_2, p_3 – principal values of a tensor of damages or their function $M(P)$, called a damage measure, where *P* – a tensor of damages with components. p_1, p_2, p_3. According to the taken forms of damages from shift and separation in the elementary case, it is possible to bring their measures

$$M_1(P) = p_1 - p_3 ; M_2(P) = p_1 \tag{10}$$

We will designate tensors *P* at the time of destruction through P_s ($p_\perp 12 = 1; p_\perp ij = 0; i,j \neq 1,2$) and P_r in a case of pure shift ($p_\perp 11 \equiv p_\perp 1 = 1; p_\perp 22 \equiv p_\perp 2 = p_\perp 33 \equiv p_\perp 3 = p_\perp r; p_\perp ij = 0; i \neq j$) , also we will receive rated criteria of destruction:

$$M_1(P) \leq M_1(P_s); M_2(P) \leq M_2(P_r)). \tag{11}$$

We will enter the first invariants and deviators of tensors:

$$\theta = \varepsilon_{kk};\ e_{ij} = \varepsilon_{ij} - \frac{1}{3}\theta\delta_{ij};\ 3\pi = p_{kk};\ \pi_{ij} = p_{ij} - \pi\delta_{ij} \tag{12}$$

where $\delta_{ij} = 1$ при $i = j$ и $\delta_{ij} = 0$ при $i \neq j$

Then the tensor (9) will break up into two:

$$\pi_{ij} = \int_0^t \psi(t-\tau)de_{ij}(\tau), \qquad \pi = \int_0^t \psi_1(t-\tau)d\theta(\tau) \tag{13}$$

and

$$\psi_1 = \frac{1}{3}\psi + \psi_2. \tag{14}$$

We will make for a uniform tubular sample the twisting effort with a constant speed of deformation

$$e_{12} = \vartheta_s t, \tag{15}$$

where $\vartheta_s = const;\ e_{ij} = 0;\ i,j = 1,2,$ and bringing him to destruction, according to (13) we will receive:

$$1 = \vartheta_s \int_0^{\tau_s} \psi(t-\tau)d\tau \tag{16}$$

If to make a series of similar experiments at various speeds of deformation, then it is possible to receive dependence $t_s = t_s(\vartheta_s)$, where t_s – time before destruction, or the return to her dependence $\vartheta_s = \vartheta_s(t_r)$. Substituting in (16) and differentiating on t_s, we find:

$$\psi(t) = \frac{\dot{\vartheta}_s(t)}{[\vartheta_s(t)]^2} = \frac{1}{S_s(t)}. \tag{17}$$

Conditions of stretching of a uniform cylindrical sample with a constant speed of deformation

$$\varepsilon_{11} \equiv \varepsilon_1 = \vartheta_r t; \quad \varepsilon_{22} = \varepsilon_{33} = -\nu\varepsilon_1; \quad \varepsilon_{ij} = 0; \quad i \neq j \; , \tag{18}$$

where ν- Poisson's coefficient. On this basis, making a series of experiments at various speeds, we receive dependence $\vartheta_r = \vartheta_r(t_r)$, here t_r – a time before destruction.

Then similarly from expression (9) dependences turn out

$$\frac{p_r}{S_r(t)} = -\nu\psi(t) + (1-2\nu)\psi_2(t); \tag{19}$$

where

$$S_r = \frac{[\vartheta_r(t)]^2}{\dot{\vartheta}_r(t)}.$$

Dependences on (17), (19) taking into account a condition (14) give $(1-2\nu)\psi_2(t) = \frac{1}{S_r(t)} - \frac{1}{S_s(t)}$

$$(1-2\nu)\psi_1(t) = \frac{1-2\nu}{3S_s(t)} + \frac{1}{S_r(t)} - \frac{1}{S_s(t)}; \tag{20}$$

And also a condition of functions similarity

$$S_r(t) = \frac{1-p_r}{1+\nu} S_s(t) \tag{21}$$

If experiences don't confirm this condition, then the measure $M_2(P)$ at simple stretching don't depend on $p_2 = p_3 = p_r$. Substituting (17) and (20) in (9), we receive dependence of a tensor of damages on the process of deformations and functions $S_s(t)$ and $S_r(t)$, the samples found in experiments on destruction at constant speeds of deformation:

$$p_{ij} = \int_0^t \frac{d\varepsilon_{ij}(\tau)}{S_s(t-\tau)} + \frac{\delta_{ij}}{1-2\nu}\int_0^t \left[\frac{1}{S_r(t-\tau)} - \frac{1}{S_s(t-\tau)}\right] d\varepsilon_{kk}(\tau) \tag{22}$$

From conditions (11) where maxima of measures of damage on body coordinates appear, there is time before its final fracture. The concept of a measure of damage can be used for direct modeling of the process of destruction on models even from other material according to a condition

$$[M(P)]_{м} = [M(P)]_{н} \tag{23}$$

where index «м» belongs to a model, and index «н» belongs to a natural body what in its turn requires equality of components of a tensor of damages to model and in a natural body of rather dimensionless time t/t_0, here t_0 – characteristic time. Definition of time before destruction from conditions (11) on the true deformations found by photoelasticity method and in experiments on the destruction of natural samples to functions $S_s(t)$ and $S_r(t)$ doesn't make basic difficulties.

CONCLUSIONS

Thus, the damageability tensor on the basis of deformations where true deformations at the accepted level of the macroscopic description of the structure of the material are offered to be determined by a polarizing and optical method with an observance of conditions of similarity of deformation processes is constructed. On experimentally found true deformations and functions received from creep experimentsitispossible to define the time before destruction.

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